# CSE222 HW5

## Q1

1. At every depth there exists nodes. Therefore, total depth of nodes at the same depth is . To find the total depth of the complete binary tree we must sum this for every depth. Therefore, total depth of a complete binary tree is:  
   With the recurrence relation is solved as:
2. With a complete binary tree length of half of its nodes are at the bottom. So, when a random value that is inside the tree is given its probability of being at the bottom is while its probability of being at depth is . For a randomly selected node value in a complete tree with depth its probability of being at depth is . If it’s found at depth then that many comparisons have been made. So, the average number of comparisons for successful search is:  
   We know that for a complete binary tree its depth is , assuming that it has elements. So, for number of elements the equation becomes:
3. If a leaf node is turned into an inner node with two leaf nodes the total change in number of leaf nodes is plus one. We know that for a full tree with one inner node there is two leaf nodes. Now assume to be the function that gives number of leaf nodes for many inner nodes. Assume that the function holds for . Now change one of the leaf nodes to an inner node, making , function becomes:  
   The function holds the one increment of number of leaf nodes upon one increment of number of inner nodes, therefore it is proven that for leaf nodes making that number of total nodes has to be . So, there can’t be a full binary tree with even number of nodes. For a full tree with n elements there are .

## Q2

